Linnæus University

Jnr: 2021/3630-3.1.2.2

Course syllabus

Faculty of Technology
Department of Mathematics

4MA501 Sannolikhetsteorins matematiska grunder, 7,5 högskolepoäng

Foundations of probability, 7.5 credits

Main field of study
Mathematics

Subject Group
Mathematics

Level of classification
Second Level

Progression
A1N

Date of Ratification
Approved 2014-10-03
Revised 2021-09-30 by Faculty of Technology. Revision of prerequisites, content, literature and adjustment of goals.
The course syllabus is valid from autumn semester 2022

Prerequisites
General entry requirements for secondcycle studies and specific entry requirements:

- 1MA501 Probability Theory and Statistics 7.5 credits,
- 1MA465 Multivariable Calculus and Vector Calculus 7.5 credits, or the equivalent.

Objectives

After completing the course, students should be able to
1. independently and with adequate techniques solve problems, perform calculations and conduct lines of reasoning within the part of foundations of probability covered by the course and writing to communicate these solutions, calculations and reasoning; in particular

- master the basic tools in the form of structures and methods needed to describe the measure-theoretical and probabilistic concepts
- apply the above tools in a given situation
- defend the choice of these tools in a coherent and precise manner
- calculate the characteristic quantities of a probability distribution and interpret them
- evaluate the selected tools in connection with a given problem and to select the most appropriate
- master the basic tools in the form of structures and methods needed to describe the measure-theoretical and probabilistic concepts

- apply the above tools in a given situation
- defend the choice of these tools in a coherent and precise manner
- calculate the characteristic quantities of a probability distribution and interpret them
- evaluate the selected tools in connection with a given problem and to select the most appropriate

2. verbally conduct lines of reasoning in the part of foundations of probability theory covered by the course; in particular

- formulate and prove the probability theory claims to formulate and prove theorems in measure theory and integration theory
- interpret, communicate and argue the basic content of the measure and advanced probability theory

3. implement a graded assignment report, in particular

- present the graded report in a concise manner
- master the basic tools in the form of structures and methods needed to resolve the graded assignment
- apply the above tools for the graded assignment
- defend the choice of these tools in a coherent and precise manner
- evaluate the chosen tools of the graded assignment and to select the most appropriate
- independently obtain knowledge of the graded assignment by consulting relevant literature

Content

The course objective is to introduce the student to the central results of measure theory, integration theory and probability theory:

- sigma-algebras and Dynkin system
- measurability and measure spaces
- zero-continuity and completeness of measures
- inferior and superior limes of events
- Kolmogorov's axioms
- random variables
- independence of events and independence of random variables
- conditional probabilities and Bayes' formula
- simple and elementary functions, Lebesgue integrals and Stieljes integrals
- characterization and properties of distribution functions
- the concept of bounded convergence and Fatou's lemma
- moments and characteristic functions of random variables and their properties
- probabilities on product space as well as marginal and conditional distributions
- covariance and correlation and their properties
- fundamental moment inequalities (Hölder, Minkowski, Markov, Cantelli)
- Kolmogorov's 0-1 law and Borel Cantelli's lemmas
- different convergence notions of random variables and their relations
- laws of large numbers and the central limit theorem.
- introduction to martingale theory.

Type of Instruction

Lectures and exercises

Examination

The course is assessed with the grades A, B, C, D, E, Fx or F.
The grade A constitutes the highest grade on the scale and the remaining grades follow in descending order where the grade E is the lowest grade on the scale that will result in

a pass. The grade F means that the student's performance is assessed as fail (i.e. received the grade F).

Assessment of how well the student fulfills the objectives is achieved through

- graded exercises
- oral examination

Repeat examination is offered in accordance with Local regulations for courses and examination at the first and second-cycle level at Linnaeus University.

If the university has decided that a student is entitled to special pedagogical support due to a disability, the examiner has the right to give a customised exam or to have the student conduct the exam in an alternative way.

Course Evaluation

During the implementation of the course or in close conjunction with the course, a course evaluation is to be carried out. Results and analysis of the course evaluation are to be promptly presented as feedback to the students who have completed the course. Students who participate during the next course instance receive feedback at the start of the course. The course evaluation is to be carried out anonymously.

Credit Overlap

The course cannot be included in a degree along with the following courses of which the content fully, or partly, corresponds to the content of this course: 4MA201 Foundations of probability, 7.5 credits

Other

Grade criteria for the A–F scale are communicated to the student through a special document. The student is to be informed about the grade criteria for the course by the start of the course at the latest.

Required Reading and Additional Study Material

Gut, Allan, *Probability: A Graduate Course,* Springer Texts in Statistics, latest edition, 603 p